Accounting for magnetic saturation in induction machines modelling

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Abstract: A new model is developed for (uniform air-gap) induction motor that accounts for the saturation feature of the machine magnetic circuit. It is built-up starting from the basic electrical/mechanical laws and turns out to be quite different from the standard model that is widely used in control oriented literature. Specifically, the new model involves state-dependent parameters. An experimental validation using a real 7.5 kW machine proved the good accuracy of the proposed model.

Keywords: induction motor; magnetic saturation; modelling; identification; control.

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1 Introduction

The problem of induction motor control and observation has been given a great deal of interest over the last decade (see, e.g., Ortega et al., 1996; Caron and Hautier, 1995; Hu et al., 1996; Belkheiri and Fares, 2008; Bouchhida et al., 2008). However, most of the previous works have been based on the standard model where the magnetic characteristic is described by a linear relation. As a matter of fact, such a characteristic is non-linear in physical machines: it exhibits saturation and hysteresis features. To achieve high-performance varying-speed operation mode for induction machines, it is necessary to use controllers that allow large flux variations. Indeed, allowing large flux variations makes possible the achievement of a suitable power factor and a high efficiency (by limiting current absorption). But, in order to allow large flux variations the controller must be developed by using a model that takes into account the non-linear nature of the machine magnetic characteristic. The question is: how such model can be obtained?

The present work precisely focuses on such a modelling issue. This has been coped with (Sullivan and Sanders, 1992; Novotnak, 1995; Pedra et al., 2009) by just letting the mutual inductance coefficient associated with a given rotor flux direction (d or q) be a non-linear function of the stator current along the same direction. This assumption is not realistic because it amounts to neglect the cross-saturation feature. Furthermore, neither the non-linear functions approximating the mutual inductances nor the resulting (saturated) model are explicitly described in the mentioned works. The present paper develops a new approach that appropriately accounts for the magnetic saturation phenomenon in induction machine modelling. It thoroughly contrasts with the previous approaches which heavily relied upon the standard unsaturated model. Presently, the starting point is the machine electric equivalent scheme where the stator and the rotor are represented by triphase coils (Leonhard, 2001). Additional physical laws will be applied to account for the (well-known) coupling that exists (even in the case of a uniform air-gap machine) between both axes of an AC-machine (Levi, 1997). Such a coupling (usually called cross-saturation), is due to the non-linear properties of magnetic materials. As suggested in Garrido et al. (1995), the magnetic characteristic can be approximated by a non-linear function that could be polynomial, exponential, arctangent, etc. Such a function links the air-gap flux Φ_μ to the magnetising current I_μ (this includes the contribution of both stator and rotor currents). The obtained model is experimentally validated using a 7.5 kW AC-machine (see Ouadi, 2004). The input signals are chosen so that the machine operates both in the linear and non-linear parts of the corresponding magnetic characteristic. The resulting responses of the new model turn out to be sufficiently close to those of the true machine. On the contrary, the standard model responses are not so close, especially when the machine operates in the saturation part.

The paper is organised as follows: Section 2 is devoted to modelling the magnetic characteristic of the studied machine; the remaining electromechanical equations are established in Section 3; in Section 4, the static magnetisation parameter is introduced and in Section 5, machine modelling is completed by establishing its state-space representation; an experimental validation of the obtained model as well as a comparison with the standard model are performed in Section 6.

Table 1 Notations

R_s, R_r	Stator and rotor resistances
l_s, l_r	Stator and rotor leakage inductances (constant parameters)
ϕ_μ	Magnetising flux at a stator phase (flux within the air-gap)
Φ_μ	Norm of the flux ϕ_μ
ϕ_s	Stator flux (through one phase)
ϕ_r	Rotor flux (through one phase)
i_s, i_r	Stator and rotor currents (through one phase)
$i_r^{'}$	Current in a rotor phase brought back to the stator, $i_r^{'} = i_r e^{j\theta}$
I_μ	Magnetising current $i_\mu = i_s + i_r^{'}$; I_μ is its norm
k_s, k_r	Coefficients of the stator and rotor coils, respectively
n_s, n_r	Number of conductors beneath each stator pole (resp. rotor pole)
Ω, ω_s	Machine angular speed and stator current frequency, respectively
ω_r	Rotor current frequency ($\omega_r = \omega_s - \omega$)
Ω	Rotation speed of the machine rotor, one has $w = p\Omega$.
θ, θ_s	Angular positions of the rotor and rotating field, respectively
T_e, T_L	Electric motor torque and load torque, respectively
J	Rotor inertia
P	Number of poles pairs
$[v_s]_{123}$ $[v_r]_{123}$	Triphase stator and triphase rotor voltage, respectively

Table 1 Notations (continued)

$[\phi_s]_{123}$ $[\phi_r]_{123}$	Triphase stator and triphase rotor flux, respectively
$[i_s]_{123}$ $[i_r]_{123}$	Triphase stator and triphase rotor current, respectively
$[v_{sd}\ v_{sq}]$ $[v_{rd}\ v_{rq}]$	(d, q) components of stator and rotor voltages
$[i_{sd}\ i_{sq}]$ $[i_{rd}\ i_{rq}]$	(d, q) components of stator and rotor currents
$[\phi_{sd}\ \phi_{sq}]$ $[\phi_{rd}\ \phi_{rq}]$	(d, q) components of stator and rotor fluxes
$[\phi_{\mu d}\ \phi_{\mu q}]$	(d, q) components of the magnetising flux
$P_k(\Psi)$	Park transformation (Ψ is its angle)
$k = \dfrac{k_r n_r}{k_s n_s}$	Transformation ratio of the AC machine

2 Characterisation of magnetic saturation in AC machines

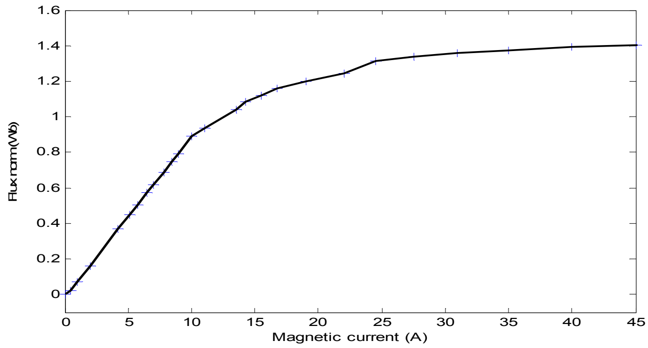
The non-linear feature of the magnetic circuit in induction motors has been accounted for in many ways. Early solutions suggested capturing this through non-linear approximations of the (B, H) characteristic (Leonhard, 2001; Seguier and Notelet, 2005). In more recent works, the magnetic saturation is accounted for through a flux-current relation called magnetic characteristic. This links the magnetising flux norm (i.e., the useful flux at a stator phase) to the magnetising current (Levi, 1997). Several laws have been suggested to describe the magnetic characteristic, e.g.,:

$$\Phi_\mu = \alpha\beta^{I_\mu} I_\mu^\gamma \text{ or } I_\mu = \frac{s_1 - s_2}{(b^{-n} + \Phi_\mu^{-n})^{1/n}} + s_2\Phi_\mu.$$

In the present work, a spline polynomial approximation is developed to approach the non-linearity of the magnetic circuit. Let the (unknown) real magnetic characteristic be denoted $I_\mu = \lambda(\Phi_\mu)$. Our purpose is to build up a polynomial approximation for $\lambda(I_\mu)$. The first step consists in obtaining experimentally a set of points of the real machine characteristic. A spline interpolation of the experimental points is then performed, using suitable tools, to get a polynomial approximation, denoted $P(.)$ of the unknown function $\lambda(.)$. The larger the degree of $P(.)$, the more smooth and accurate the approximation. Figure 1 shows the polynomial $P(.)$ obtained with $n = 8$. In the sequel, we will also need an approximation of the inverse characteristic $I_\mu = \lambda^{-1}(\Phi_\mu)$. A polynomial approximation $P_{inv}(.)$ is obtained directly from the available experimental set of points, using the same tools as previously (Figure 2).

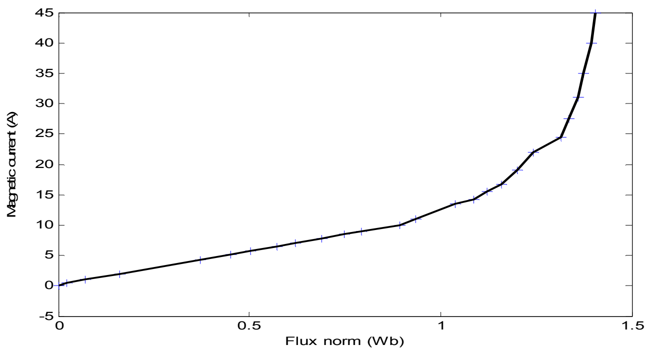
Remark 1. Figure 1 shows that the largest linear zone of the magnetic characteristic corresponds to small values of the flux ($\Phi_\mu < 0.7$ *Wb*) whereas the machine nominal point, presently equal to $\Phi_\mu = 0.9$ *Wb*, is located at the saturation elbow.

Figure 1 Magnetic characteristic $\Phi_\mu = \lambda(I_\mu)$ (see online version for colours)



Notes: Crosses: experimental points (I_μ, Φ_μ),
solid: interpolation $P(.)$, unities: I_μ (A), Φ_μ (Wb)

Figure 2 Inverse magnetic characteristic $I_\mu = \lambda^{-1}(\Phi_\mu)$ (see online version for colours)



Notes: Crosses: experimental points (Φ_μ, I_μ),
solid: spline interpolation $P_{inv}(.)$,
unities: I_μ (A), Φ_μ (Wb)

3 Induction machines equations

3.1 Park transformation of the stator and rotor voltages

The modelling is based on standard assumptions i.e., the machine is symmetrical, the air gap is smooth, the ferromagnetic losses are negligible, the induction distribution through the air gap is sinusoidal and all electromagnetic variables ($[\phi_s]_{1,2,3}$, $[\phi_r]_{1,2,3}$, $[i_s]_{1,2,3}$,…) define a well balanced triphase system, i.e., $\phi_{s1} + \phi_{s2} + \phi_{s3} = 0$. Applying the Park transformation to the triphase equations yields the following electrical equations, completed by the mechanical equation (see e.g., Leonhard, 2001 and Seguier and Notelet, 2005):

$$V_{sd} = R_s.i_{sd} + \frac{d}{dt}\phi_{sd} - \omega_s\phi_{sq} \tag{1}$$

$$V_{sq} = R_s.i_{sq} + \frac{d}{dt}\phi_{sq} + \omega_s\phi_{sd} \tag{2}$$

$$0 = R_r.i_{rd} + \frac{d}{dt}\phi_{rd} - (\omega_s - \omega)\phi_{rq} \tag{3}$$

$$0 = R_r .i_{rq} + \frac{d}{dt}\phi_{rq} + (\omega_s - \omega)\phi_{rd} \quad (4)$$

$$\frac{d\Omega}{dt} = \frac{T_e}{J} - \frac{T_L}{J} - \frac{f}{J}\Omega \quad (5)$$

3.2 Saturated flux equations in the (*d*, *q*) coordinates system

The magnetic fluxes for the stator and rotor phases are given by (see Leonhard, 2001 and Seguier and Notelet, 2005):

$$\phi_s = \phi_{leakage/st} + \phi_\mu = l_s i_s + \phi_\mu \quad (6)$$

$$\phi_r = \phi_{leakage/rt} + \phi_{\mu r} = l_r i_r + \phi_{\mu r} \quad (7)$$

where ϕ_μ and $\phi_{\mu r}$ respectively denote the magnetising air-gap fluxes along one phase of the stator and rotor; $\phi_{leakage/st}$ and $\phi_{leakage/rt}$ respectively denote the stator and rotor leakage fluxes. Equalities (6) to (7) mean that $\phi_{leakage/st}$ and $\phi_{leakage/rt}$ are respectively proportional to the stator and rotor currents. This is valid in real machines because the leakage flux, circulating in air, not in iron, is not large. Let k denote the machine transformation ratio. The flux equations (6) to (7) become, in the (d, q) coordinates:

$$\phi_{sd} = l_s i_{sd} + \phi_{\mu d} \quad (8)$$

$$\phi_{sq} = l_s i_{sq} + \phi_{\mu q} \quad (9)$$

$$\phi_{rd} = l_r i_{rd} + k\phi_{\mu d} \quad (10)$$

$$\phi_{rq} = l_r i_{rq} + k\phi_{\mu q} \quad (11)$$

The contribution of the stator and rotor in the air-gap flux generation is expressed in term of the magnetising current, denoted i_μ (Leonhard, 2011; Levi, 1997). Therefore, the ($d - q$) components of the $[i_\mu]_{123}$ system satisfy:

$$i_{\mu d} = i_{sd} + k\, i_{rd} \text{ and } i_{\mu q} = i_{sq} + k\, i_{rq} \quad (12)$$

The non-linear feature of the machine magnetic characteristic causes cross-saturation effect (Levi, 1997; Vagati et al., 2000) i.e., the component of a given (stator, rotor or magnetising) flux, along a given (d or q) axis is dependent on both the d- and q-components of both stator and rotor currents. Inspired from Garrido et al. (1988), the cross-saturation effect is accounted for letting the magnetising flux be expressed as follows:

$$\phi_{\mu d} = M_d i_{\mu d} + M_{dq} i_{\mu q} + \phi_{d0} \quad (13)$$

$$\phi_{\mu q} = M_q i_{\mu q} + M_{dq} i_{\mu d} + \phi_{q0} \quad (14)$$

where M_d, M_q and M_{dq} are new inductive parameters and:

$i_{\mu d}$, $i_{\mu q}$ denote the (d, q) components of the magnetising current

$M_d\, i_{\mu d}$ is the magnetising flux generated, along the d-axis, by the components (along the same axis) of the stator and the rotor currents

$M_q\, i_{\mu q}$ is the magnetising flux generated, along the q-axis, by the components (along the same axis) of the stator and the rotor currents

$M_{dq}\, i_{\mu d}$ is the coupling magnetising flux generated, along the q-axis, by the components (along the same axis) of the stator and the rotor currents

$M_{dq}\, i_{\mu q}$ is the coupling magnetising flux generated, along the d-axis, by the components (along the same axis) of the stator and the rotor currents

ϕ_{d0}, ϕ_{q0} are additional terms whose values depend on the inductive parameters.

It is worth noticing that the coefficients M_d, M_q and M_{dq} are not uniquely defined. A judicious definition, suggested in Garrido et al. (1988), consists in choosing them so that, when differentiating (13) and (14) with respect to $i_{\mu d}$ and $i_{\mu q}$, one gets:

$$M_d \overset{def}{=} \frac{\partial \phi_{\mu d}}{\partial i_{\mu d}}; \quad M_q \overset{def}{=} \frac{\partial \phi_{\mu q}}{\partial i_{\mu q}}; \quad M_{dq} \overset{def}{=} \frac{\partial \phi_{\mu d}}{\partial i_{\mu q}} = \frac{\partial \phi_{\mu q}}{\partial i_{\mu d}} \quad (15)$$

It follows from (13) and (14) that the functions ϕ_{d0} and ϕ_{q0} must undergo the following differential equations:

$$\frac{\partial \phi_{d0}}{\partial i_{\mu d}} = -\frac{\partial M_d}{\partial i_{\mu d}} i_{\mu d} - \frac{\partial M_{dq}}{\partial i_{\mu d}} i_{\mu q}, \qquad \frac{\partial \phi_{q0}}{\partial i_{\mu d}} = -\frac{\partial M_q}{\partial i_{\mu q}} i_{\mu q} - \frac{\partial M_{dq}}{\partial i_{\mu q}} i_{\mu d} \quad (16a)$$

$$\frac{\partial \phi_{d0}}{\partial i_{\mu q}} = -\frac{\partial M_d}{\partial i_{\mu q}} i_{\mu d} - \frac{\partial M_{dq}}{\partial i_{\mu q}} i_{\mu q} \text{ and } \frac{\partial \phi_{q0}}{\partial i_{\mu d}} = -\frac{\partial M_q}{\partial i_{\mu d}} i_{\mu q} - \frac{\partial M_{dq}}{\partial i_{\mu d}} i_{\mu d} \quad (16b)$$

On the other hand, substituting (13) to (14) in (8) to (11) leads to the following expressions for the (d, q) flux coordinates:

$$\begin{aligned}
\phi_{sd} &= l_s i_{sd} + M_d i_{\mu d} + M_{dq} i_{\mu q} + \phi_{d0} \\
\phi_{sq} &= l_s i_{sq} + M_q i_{\mu q} + M_{dq} i_{\mu d} + \phi_{q0} \\
\phi_{rd} &= l_r i_{rd} + k\left(M_d i_{\mu d} + M_{dq} i_{\mu q} + \phi_{d0}\right) \\
\phi_{rq} &= l_r i_{rq} + k\left(M_q i_{\mu q} + M_{dq} i_{\mu d} + \phi_{q0}\right)
\end{aligned} \quad (17)$$

4 Relation between the induction coefficients and the static magnetisation parameter

4.1 Definition of static magnetisation parameter

The non-linear feature of the machine magnetic circuit is entirely accounted for through the magnetic characteristic $\Phi_\mu = \lambda(I_\mu)$ (Subsection 2.1). As, the instantaneous quantities ϕ_μ and i_μ are synchronous, one has the relation:

$$\phi_\mu = \frac{\lambda(I_\mu)}{I_\mu} i_\mu \tag{18}$$

The forthcoming development involves the static magnetising parameter m, see [5]:

$$m = \frac{\Phi_\mu}{I_\mu} = \frac{\lambda(I_\mu)}{I_\mu} \overset{\text{def}}{=} \mathrm{h}(I_\mu) \tag{19}$$

In view of (18), the (d, q)-components of the magnetising flux can be expressed as follows:

$$\phi_{\mu d} = m\, i_{\mu d}, \quad \phi_{\mu q} = m\, i_{\mu q} \tag{20}$$

Then, the expressions (8)–(11) of the stator and rotor flux become:

$$\begin{aligned} \phi_{sd} &= l_s i_{sd} + m\, i_{\mu d}, \quad & \phi_{sq} &= l_s i_{sq} + m\, i_{\mu q} \\ \phi_{rd} &= l_r i_{rd} + k\, m\, i_{\mu d}, \quad & \phi_{rq} &= l_r i_{rq} + k\, m\, i_{\mu q} \end{aligned} \tag{21}$$

More generally, it is shown in subsequent sections, that all machine parameters, including induction coefficients, can be expressed as a function of m. But first, let us rewrite m as a function of the machine state variables. Indeed from (19) and (17), one gets:

$$\begin{aligned} m &= \frac{\sqrt{\phi_{\mu d}^2 + \phi_{\mu q}^2}}{\lambda^{-1}\left(\sqrt{\phi_{\mu d}^2 + \phi_{\mu q}^2}\right)} \\ &= \frac{\sqrt{\left(\phi_{sd} - l_s i_{sd}\right)^2 + \left(\phi_{sq} - l_s i_{sq}\right)^2}}{\lambda^{-1}\left(\sqrt{\left(\phi_{sd} - l_s i_{sd}\right)^2 + \left(\phi_{sq} - l_s i_{sq}\right)^2}\right)} \end{aligned} \tag{22}$$

4.2 Analytical expressions of induction coefficients

The non-linearity of the magnetic characteristic implies that the induction parameters M_d, M_q and M_{dq} are varying with the state variables. As the model involves such parameters, these need to be computed online. This makes it necessary to explicitly express such parameters in terms of the machine state variables. One has from (15) and (20):

$$M_d = m + \frac{\partial m}{\partial i_{\mu d}} i_{\mu d} = m + \frac{dm}{dI_\mu}\frac{i_{\mu d}^2}{I_\mu} \tag{23}$$

$$M_q = m + \frac{dm}{dI_\mu}\frac{i_{\mu q}^2}{I_\mu}, M_{dq} = \frac{dm}{dI_\mu}\frac{i_{\mu d} i_{\mu q}}{I_\mu} \tag{24}$$

using the fact that $I_\mu^2 = i_{\mu d}^2 + i_{\mu q}^2$. Now, let us build up mathematical expressions that explicitly link the induction coefficients M_d, M_q and M_{dq} to the state variables. Equations (23) to (24) show that this objective can be reached by expressing m and $dm\,/dI_\mu$ in terms of the machine state variables (measured or observed). First, recall that a set of experimental points $(I_\mu,\Phi_\mu) = [I_\mu, \lambda(I_\mu)]$ is available. A set of experimental couples $[I_\mu, h(I_\mu)]$ can thus be readily obtained, where $h(.)$ is as in (19). The experimental couples can be used to get a smooth polynomial approximation of the function $h(.)$. The obtained approximation for the considered machine, denoted $Q(.)$, is represented by Figure 3. Notice also that the function $Q(.)$ is time-derivable and bounded away from zero.

On the other hand, as expressions (23) to (24) involve the derivative $dm\,/dI_\mu$, it also has to be approximated by a polynomial, denoted $R(I_\mu)$. The obtained function $R(I_\mu)$ is shown in Figure 4. The approximations obtained so far are summarised in Table 2 and will now be based upon to get expressions that explicitly link the induction coefficients to the state variables. To this end, it follows from (19) and (22) that:

$$\Phi_\mu = \sqrt{(\phi_{sd} - l_s i_{sd})^2 + (\phi_{sq} - l_s i_{sq})^2} \tag{25}$$

$$I_\mu = P_{inv}\left(\sqrt{(\phi_{sd} - l_s i_{sd})^2 + (\phi_{sq} - l_s i_{sq})^2}\right) \tag{26}$$

Therefore, combining equations (23)–(24) with (21) leads to the following expressions:

$$M_d = Q(I_\mu) + R(I_\mu)\frac{(\phi_{sd} - l_s i_{sd})^2}{I_\mu Q^2(I_\mu)} \tag{27}$$

$$M_q = Q(I_\mu) + R(I_\mu)\frac{(\phi_{sq} - l_s i_{sq})^2}{I_\mu Q^2(I_\mu)} \tag{28}$$

$$M_{dq} = R(I_\mu)\frac{(\phi_{sd} - l_s i_{sd})(\phi_{sq} - l_s i_{sq})}{I_\mu Q^2(I_\mu)} \tag{29}$$

Figure 3 Experimental couples (I_μ, m) (crosses) and spline interpolation $Q(I_\mu)$ (solid), unities: I_μ (A), m (Wb/A) (see online version for colours)

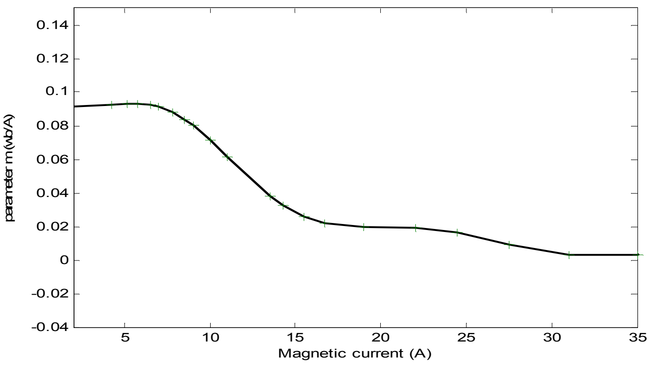
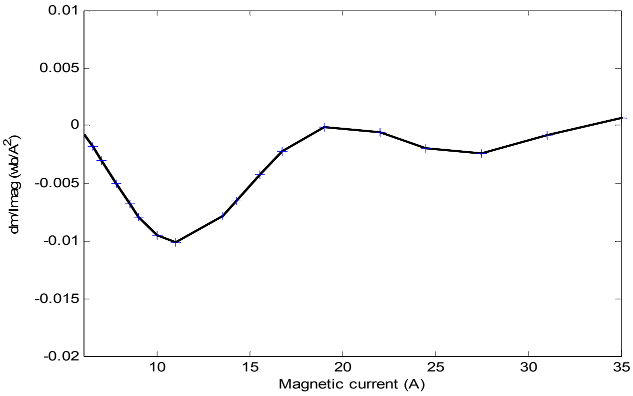


Figure 4 Experimental couples I_μ, dm / dl_μ (crosses), spline interpolation $R(I_\mu)$ (solid), unities: I_μ (A), dm / dl_μ (Wb/A^2) (see online version for colours)



5 Induction machine model development accounting for magnetic saturation feature

5.1 Rotor flux equations

From (3) one readily gets that:

$$\frac{d\phi_{rd}}{dt} = -R_r i_{rd} + \omega_r \phi_{rq} \tag{30}$$

Since i_{rd} is not a state variable, it should be removed from the above equation. To this end, one obtains from (21) and (12) that:

$$i_{rd} = \frac{\phi_{rd} - k\, m\, i_{sd}}{l_r + k^2 m} \tag{31}$$

which together with (30), yields:

$$\frac{d\phi_{rd}}{dt} = -g_1 \phi_{rd} + g_2 i_{sd} + \omega_r \phi_{rq} \tag{32}$$

with $g_1 = \dfrac{R_r}{l_r + k^2 m}$ and $g_2 = \dfrac{R_r km}{l_r + k^2 m}$. This is the first state equation. To obtain the second one, let (4) be rewritten as follows:

$$\frac{d\phi_{rq}}{dt} = -R_r . i_{rq} - \omega_r \phi_{rd} \tag{33}$$

Since i_{rq} is not a state variable it should be removed from the above equation. To this end, one gets from (21) and (12) that:

$$i_{rq} = \frac{\phi_{rq} - kmi_{sq}}{l_r + k^2 m} \tag{34}$$

which, together with (33), yields:

$$\frac{d\phi_{rq}}{dt} = -g_1 \phi_{rq} + g_2 i_{sd} - \omega_r \phi_{rd} \tag{35}$$

5.2 Stator current equations

These are derived from equations (1) to (2) which are rewritten here for convenience:

$$V_{sd} = R_s . i_{sd} + \frac{d}{dt}\phi_{sd} - \omega_s \phi_{sq} \tag{36}$$

$$V_{sq} = R_s . i_{sq} + \frac{d}{dt}\phi_{sq} + \omega_s \phi_{sd} \tag{37}$$

As ϕ_{sd} and ϕ_{sq} are not state variables, they should be removed from the above equations. To this end, one gets from equations (21) that:

$$\phi_{sd} = l_s i_{sd} + \tfrac{1}{k}(\phi_{rd} - l_r i_{rd}) \tag{38}$$

$$\phi_{sq} = l_s i_{sq} + \tfrac{1}{k}(\phi_{rq} - l_r i_{rq}) \tag{39}$$

Substituting (38) to (39) in (36) to (37), and using (31) and (34), one gets:

$$\begin{aligned} V_{sd} &= R_s i_{sd} + l_s \frac{di_{sd}}{dt} + \frac{1}{k}\frac{d\phi_{rd}}{dt} - \frac{l_r}{k}\frac{di_{rd}}{dt} \\ &\quad - \omega_s \left(l_s + \frac{l_r m}{l_r + k^2 m} \right) i_{sq} - \frac{\omega_s m}{k(l_r + k^2 m)} \phi_{rq} \end{aligned} \tag{40}$$

$$\begin{aligned} V_{sq} &= R_s . i_{sq} + l_s \frac{di_{sq}}{dt} + \frac{1}{k}\frac{d\phi_{rq}}{dt} - \frac{l_r}{k}\frac{di_{rq}}{dt} \\ &\quad + \omega_s \left(l_s + \frac{l_r m}{l_r + k^2 m} \right) i_{sd} + \frac{\omega_s m}{k(l_r + k^2 m)} \phi_{rd} \end{aligned} \tag{41}$$

In this expression, di_{rd} / dt and di_{rq} / dt must be expressed in terms of the state variables. To this end, time-derivation of the rotor flux equations (17) gives, using (16):

$$\begin{aligned} (l_r + k^2 M_d)\frac{di_{rd}}{dt} &= \frac{d\phi_{rd}}{dt} - kM_d \frac{di_{sd}}{dt} \\ &\quad - kM_{dq}\frac{di_{sq}}{dt} - k^2 M_{dq}\frac{di_{rq}}{dt} \\ (l_r + k^2 M_q)\frac{di_{rq}}{dt} &= \frac{d\phi_{rq}}{dt} - kM_q \frac{di_{sq}}{dt} \\ &\quad - kM_{dq}\frac{di_{sd}}{dt} - k^2 M_{dq}\frac{di_{rd}}{dt} \end{aligned} \tag{42}$$

Solving these equations with respect to di_{rd} / dt and di_{rq} / dt yields, using (35) and (32):

$$\begin{aligned} \frac{di_{rd}}{dt} &= a_0 \left[\frac{-R_r (l_r + k^2 M_q)}{l_r + k^2 m} + \omega_r k^2 M_{dq} \right] \phi_{rd} + \\ &\quad a_0 \left[\frac{R_r k^2 M_{dq}}{l_r + k^2 m} + \omega_r (l_r + k^2 M_q) \right] \phi_{rq} \\ &\quad - a_0 \left[\frac{R_r mk^3 M_{dq}}{l_r + k^2 m} \right] i_{sq} - a_0 \left[\frac{R_r km(l_r + k^2 M_q)}{l_r + k^2 m} \right] i_{sd} \\ &\quad - a_0 \left[kM_d (l_r + k^2 M_q) - k^2 M_{dq}^2 \right] \frac{di_{sd}}{dt} - a_0 \left[l_r kM_{dq} \right] \frac{di_{sq}}{dt} \end{aligned} \tag{43}$$

with $a_0 = \left((l_r + k^2 M_q)(l_r + k^2 M_q) - k^4 M_{dq}^2\right)^{-1}$. Substituting (43) and (32) in (40), the expression of stator voltage along the *d* axis becomes:

$$\begin{aligned} v_{sd} &= (a_1 + a_1')i_{sd} - (a_2 + a_2'' + \omega_r a_2')\phi_{rd} \\ &\quad - (a_3 - \omega_r a_3' + \omega_s a_3'')\phi_{rq} \\ &\quad + (a_4 - \omega_s a_4')i_{sq} + a_5 \frac{di_{sd}}{dt} + a_0 a_6 \frac{di_{sq}}{dt} \end{aligned} \tag{44}$$

where the meaning of the different parameters is given in Table 3.

Operating similar transformations on the stator voltage along the *q* axis, one gets:

$$\begin{aligned} v_{sq} &= \left(a_1' + b_1\right)i_{sq} + \left(a_4 - \omega_s a_4'\right)i_{sd} + \left(b_2 - a_2'' + \omega_r a_2'\right)\phi_{rq} \\ &\quad + \left(-a_3 - \omega_r b_3 + \omega_s a_3''\right)\phi_{rd} + b_5 \frac{di_{sq}}{dt} + a_6 \frac{di_{sd}}{dt} \end{aligned} \tag{45}$$

where the newly introduced coefficients are defined in Table 4.

Table 3 Parameters newly introduced in equations (43) to (44)

$a_0 = \left((l_r + k^2 M_q)(l_r + k^2 M_d) - k^4 M_{dq}^2\right)^{-1}$	$a_1 = a_0 \dfrac{R_r l_r m\left(l_r + k^2 M_q\right)}{l_r + k^2 m}$
$a_5 = l_s + a_0 l_r \left(M_d (l_r + k^2 M_q) - k^2 M_{dq}^2\right)$	$a_1' = R_s + \dfrac{R_r m}{l_r + k^2 m}$
$a_2 = -a_0 \dfrac{R_r l_r \left(l_r + k^2 M_q\right)}{k(l_r + k^2 m)}$, $a_4' = l_s + \dfrac{l_r m}{l_r + k^2 m}$	$a_4 = a_0 \dfrac{l_r R_r k^2 m M_{dq}}{(l_r + k^2 m)}$
$a_3'' = \dfrac{km}{l_r + k^2 m}$, $a_6 = a_0 M_{dq} l_r^2$	$a_2'' = \dfrac{R_r}{k(l_r + k^2 m)}$
$a_2' = a_0 l_r k M_{dq}$, $a_3 = a_0 \dfrac{l_r R_r k M_{dq}}{(l_r + k^2 m)}$	$a_3' = \frac{1}{k}\left(1 - a_0 l_r \left(l_r + k^2 M_q\right)\right)$

Table 4 Parameters newly introduced in equation (45)

$b_1 = a_0 \dfrac{R_r l_r m\left(l_r + k^2 M_d\right)}{l_r + k^2 m}$	$b_2 = a_0 \dfrac{l_r R_r (l_r + k^2 M_d)}{k(l_r + k^2 m)}$
$b_5 = l_s + a_0 l_r \left(M_q \left(l_r + k^2 M_d\right) - k^2 M_{dq}^2\right)$	$b_3 = k^{-1}\left(1 - a_0 l_r \left(l_r + k^2 M_d\right)\right)$

Table 5 Parameters newly introduced in (46) to (47)

$q_0 = \dfrac{b_5}{a_5 b_5 - a_6^2}$, $q_1' = q_0 \dfrac{b_5 a_4'}{a_6}$, $q_2' = q_0 a_4'$	$q_1 = q_0 \left(a_1 - b_1 - \dfrac{b_5 a_4}{a_6}\right)$
$q_2 = q_0 \left(a_4 - a_6^{-1} b_5 \left(a_1' - a_1\right)\right)$, $q_4'' = q_0 a_3''$	$q_3 = q_0 \left(a_2 - b_2 + a_6^{-1} b_5 a_3\right)$
$q_4 = q_0 \left(-a_0 a_3 - \left(a_6 a_0\right)^{-1} b_5 a_1'' - \left(a_0 a_2\right)\right)$	$q_3' = q_0 \left(a_6 a_0\right)^{-1} b_5 a_3'$
$q_4' = q_0 a_6^{-1} \left(a_6 b_3 - b_5 a_2'\right)$, $q_5 = q_0 a_6^{-1} b_5$	$d_2 = a_6^{-1} \left(a_4 - a_5 q_1\right)$
$d_1 = a_6^{-1} \left(a_1' - a_1 - a_5 q_2\right)$, $q_3'' = q_0 \left(a_6 a_0\right) b_5 a_3''$	$d_1' = -a_6^{-1} a_5 q_2'$
$d_2' = -a_6^{-1} \left(a_4' - a_5 q_1'\right)$, $d_3' = a_6^{-1} \left(a_2' + a_5 q_4'\right)$	$d_3 = a_6^{-1} \left(a_2'' + a_2 - a_5 q_4\right)$
$d_3'' = -a_6^{-1} a_5 q_4''$, $d_4' = -a_6^{-1} \left(a_3' - a_5 q_3'\right)$	$d_4 = a_6^{-1} \left(a_3 - a_5 q_3\right)$
$d_4'' = -a_6^{-1} \left(a_3'' - a_5 q_3''\right)$, $d_6 = q_0 a_5 a_6^{-1}$	$d_5 = a_6^{-1} \left(1 + a_5 q_5\right)$

Now the state equations of the stator currents can readily be obtained by solving equations (44) and (45) with respect to di_{sd}/dt and di_{sq}/dt. Doing so, one gets:

$$\begin{aligned}\frac{di_{sd}}{dt} = &-\left(q_1 + q_1'\omega_s\right)i_{sq} - \left(q_2 + q_2'\omega_s\right)i_{sd} \\ &-\left(q_3 + q_3'\omega_r + q_3''\omega_s\right)\phi_{rq} \\ &-\left(q_4 + q_4'\omega_r + q_4''\omega_s\right)\phi_{rd} - q_5 v_{sd} + q_0 v_{sq}\end{aligned} \tag{46}$$

$$\begin{aligned}\frac{di_{sq}}{dt} = &-\left(d_1 + d_1'\omega_s\right)i_{sd} - \left(d_2 + d_2'\omega_s\right)i_{sq} \\ &-\left(d_3 + d_3'\omega_r + d_3''\omega_s\right)\phi_{rd} \\ &-\left(d_4 + d_4'\omega_r + d_4''\omega_s\right)\phi_{qd} + d_5 v_{sd} - d_6 v_{sq}\end{aligned} \tag{47}$$

These equations introduce new coefficients whose meaning is described in Table 5.

5.3 Mechanical equation

The mechanical power P_m that induces the electromagnetic torque is given by (Leonhard, 2001):

$$p_m = \left(\phi_{sd}i_{sq} - \phi_{sq}i_{sd}\right)\frac{d}{dt}\theta_s + \left(\phi_{rd}i_{rq} - \phi_{rq}i_{rd}\right)\frac{d}{dt}\theta_r$$

Using the rotor currents expressions (31) and (34), in the flux equations (21) yields:

$$p_m = \frac{k\,m}{l_r + k^2 m}\left(\phi_{rd}i_{sq} - \phi_{rq}i_{sd}\right)\frac{d}{dt}\left(\theta_s - \theta_r\right)$$

which implies that the torque is given by: $T_e = \frac{pkm}{l_r + k^2 m}\left(\phi_{rd}i_{sq} - \phi_{rq}i_{sd}\right)$. The rotor motion equation turns out to be:

$$\frac{d\Omega}{dt} = \frac{p\,k\,m}{J\left(l_r + k^2 m\right)}\left(\phi_{rd}i_{sq} - \phi_{rq}i_{sd}\right) - \frac{T_L}{J} - \frac{f}{J}\Omega \tag{48}$$

This is the fifth (and last) state equation. AC machine model consists of equations (32), (35), (46), (47) and (48), which are, for convenience, rewritten in a more condensed form:

$$\dot{X} = f(X) + g(X)u \tag{49a}$$

$$y = h(X) = \left[\Omega,\ \phi_{rd}^2 + \phi_{rq}^2\right]^T \tag{49b}$$

with

$$X = \left[i_{sd}, i_{sq}, \phi_{rd}, \phi_{rq}, \Omega\right]^T,\ u = \left[v_{sd}, v_{sq}\right]^T,$$

$$g(X) = \begin{bmatrix} -q_5 & q_0 & 0 & 0 & 0 \\ +d_5 & -d_6 & 0 & 0 & 0 \end{bmatrix} \tag{49c}$$

$$f(X) = \begin{bmatrix} -\left(q_1 + q_1'\omega_s\right)i_{sq} - \left(q_2 + q_2'\omega_s\right)i_{sd} \\ -\left(q_3 + q_3'\omega_r + q_3''\omega_s\right)\phi_{rq} \\ -\left(q_4 + q_4'\omega_r + q_4''\omega_s\right)\phi_{rd} \\ -\left(d_1 + d_1'\omega_s\right)i_{sd} - \left(d_2 + d_2'\omega_s\right)i_{sq} \\ -d_3 + \left(d_3'\omega_r + d_3''\omega_s\right)\phi_{rd} \\ -\left(d_4 + d_4'\omega_r + d_4''\omega_s\right)\phi_{rq} \\ -g_1\phi_{rd} + g_2 i_{sd} + \omega_r\phi_{rq} \\ -g_1\phi_{rq} + g_2 i_{sq} - \omega_r\phi_{rd} \\ \frac{p\,k\,m}{J\left(l_r + k^2 m\right)}\left(\phi_{rd}i_{sq} - \phi_{rq}i_{sd}\right) - \frac{T_L}{J} - \frac{f}{J}\Omega \end{bmatrix} \tag{49d}$$

It is worth noting that, due to flux saturation, the model parameters (d_i, d_i', d_i'', q_i, q_i', q_i'') are dependent on the static magnetisation parameter m. If the magnetic saturation is ignored, all previous parameters become constant. The proposed model then reduces to the standard model which is widely used in the control literature, e.g., Ortega et al. (1996) and Hu et al. (1996).

6 Experimental validation

The experimental part of the study was performed in the Automatic Control Dept of GIPSA-Lab, using a real induction motor whose features are summarised in Table 5. The motor control inputs are the stator voltage amplitude (V_s) and frequency (ω_s). The values of these control signals are imposed through a DC/AC converter. The measured variables are the stator currents, electric power and the rotor speed. The whole system is controlled by a PC through a DSP Card. The first experimental task consists in obtaining a number of experimental points of the motor magnetic characteristic. These are used to construct the spline approximation $I_\mu = \lambda(\Phi_\mu)$ (Figure 1). Presently, one proceeds with the second task that consists in performing a practical validation of the newly developed model (49a–d) using real measurements.

Table 5 Induction motor characteristics

Power	P_N	7.5	*KW*
Nominal speed	Ω_N	1,450	*Tr / mn*
Nominal stator voltage	U_{sn}	380	*V*
Nominal stator current	I_{sn}	16	*A*
Nominal flux	Φ_{rn}	1	*Wb*
Nominal frequency	f_s	50	*HZ*
Poles pair number	p	20	

The experimental validation consists in comparing the responses of the model (49a–d) with experimental measures on the real motor excited by typical input voltages V_s for different values of ω_s. The conditions of this experiment are chosen in such a way that the machine operates in the non-linear part of the magnetic characteristic (large values of Φ_μ and I_μ). To this end, the applied input voltage V_s is a square signal switching between 220 and 265 V (Figure 5). The stator voltage frequency is ω_s = 200(rd/s) and the load T_L is large (T_L = 30 Nm). The obtained current norm I_s, for the real machine and the model, is shown in Figure 7. It is clearly seen that, again, the model is sufficiently accurate in representing the real machine. The corresponding static induction parameter *m* is represented by Figure 8. The initial and final values of *m* are very different (relative variation 60%). Consequently, the proposed model cannot reduce, in these conditions, to the usual standard model.

Figure 5 Second experiment: the stator voltage amplitude *Vs* (*V*) (see online version for colours)

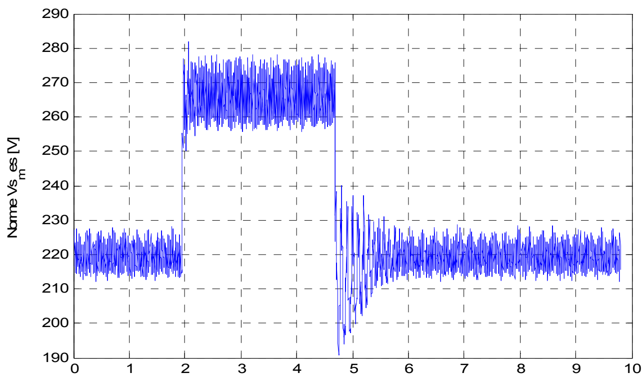
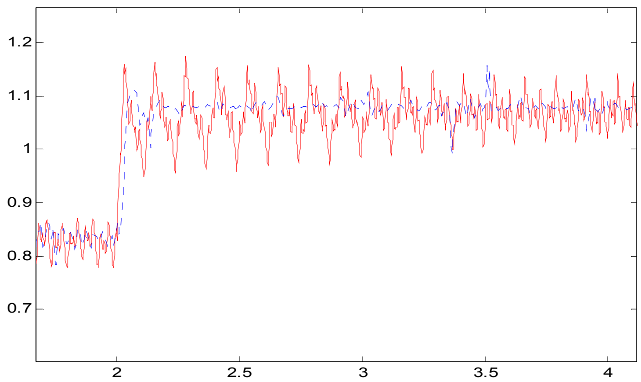
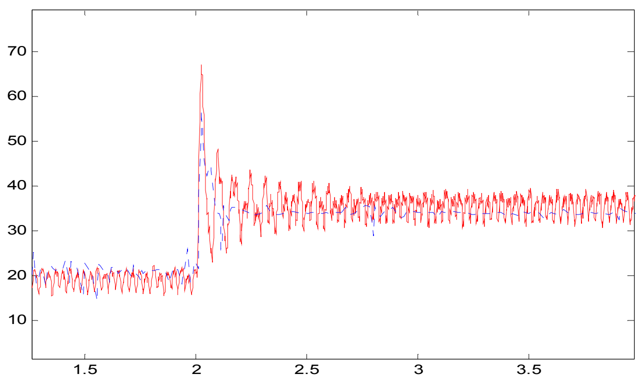


Figure 6 Second experiment: flux amplitude *Φr* (*Wb*) (see online version for colours)



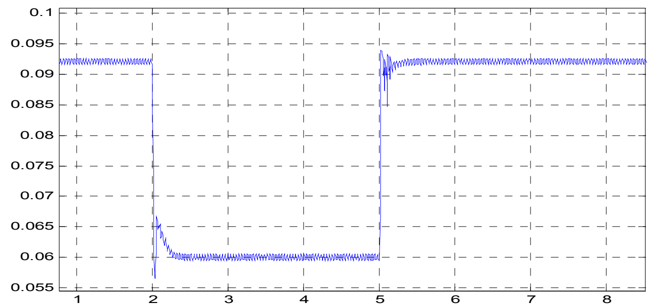
Notes: real machine (solid), model (dashed)

Figure 7 Second experiment: stator current norm I_s(*A*) (see online version for colours)



Notes: real machine (solid); model (dashed)

Figure 8 Second experiment: static induction parameter *m* (in *Wb/A*) (see online version for colours)



7 Conclusions

In this paper, a new model (49a–d) that accounts for magnetic saturation has been developed for (uniform air-gap) induction motors. Its practical validation has been performed on a 7.5 kW induction motor. Different experiments have proved that the new model is well representative of the true machine.

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